$$\frac{1}{2m-1}(a^{2m-1}+b^{2m-1}+c^{2m-1})=\mp q^{m-2}r,$$

$$\tfrac{1}{2}(a^2+b^2+c^2)=-q.$$

$$\therefore \frac{a^{2m+1}+b^{2m+1}+c^{2m+1}}{2m+1}=\frac{a^{2m-1}+b^{2m-1}+c^{2m-1}}{2m-1}\cdot\frac{a^2+b^2+c^2}{2}.$$

When $m=2$, 3, we get the results in the problem.

(2). Similarly, $(1+ax)(1+bx)(1+cx)(1+dx)=1+qx^2+rx^3+sx^4$.

$\therefore \dfrac{(-1)^{n-1}}{n}(a^n+b^n+c^n+d^n)$ is equal to the coefficient of x^n in

$$(qx^2+rx^3+sx^4)-\tfrac{1}{2}(qx^2+rx^3+sx^4)^2+\ldots\ldots\pm(1/n)(qx^2+rx^3+sx^4)^n.$$

$\therefore$ As before

$$\frac{a^{2m+1}+b^{2m+1}+c^{2m+1}+d^{2m+1}}{2m+1}=\frac{a^{2m-1}+b^{2m-1}+c^{2m-1}+d^{2m-1}}{2m-1}\cdot\frac{a^2+b^2+c^2+d^2}{2}.$$

The same reasoning will lead to the following :

$$\frac{a^{2m+1}+b^{2m+1}+\ldots\ldots+k^{2m+1}}{2m+1}=\frac{a^{2m-1}+b^{2m-1}+\ldots\ +k^{2m-1}}{2m-1}\cdot\frac{a^2+b^2+\ldots\ldots+k^2}{2}.$$

Also solved by *J. M. BOORMAN*, *J. SCHEFFER*, and the *PROPOSER*.

GEOMETRY.

139. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

If $x^2+y^2=1$ [x and y being points corresponding to complex numbers], prove that x and y are at the ends of conjugate radii of an ellipse whose foci are ±1. [From *Harkness and Morley's Introduction to the Theory of Functions.*]

Solution by J. W. YOUNG, Oliver Fellow in Mathematics, Cornell University, Ithica, N. Y., and FRANK GIFFIN, Assistant in Mathematics, University of Colorado, Boulder, Col.

Let $x=h+ik$, $y=m+in$.

The condition $x^2+y^2=1$, gives on equating real and imaginary parts,

$$h^2+m^2-k^2-n^2=1\ldots.(1),\ hk+mn=0\ldots.(2).$$

Now if the points (h, k) and (m, n) are the extremities of conjugate radii of an ellipse, we may write

$$\begin{matrix} h=a\cos\phi \\ k=b\sin\phi \end{matrix}\ \left\{\begin{matrix} m=a\sin\phi \\ n=-b\cos\phi \end{matrix}\right\}\ (a,\ b \text{ semi-axes of ellipse}).$$

These values satisfy condition (2).

Substituting in (1) we have

$$a^2-b^2=1\ldots.(3).$$

If e is the eccentricity of the ellipse $b^2=a^2(1-e^2)$, substituting in (3) and reducing, we have $ae=\pm1$, *i. e.* the foci of the ellipse are at the points ±1.

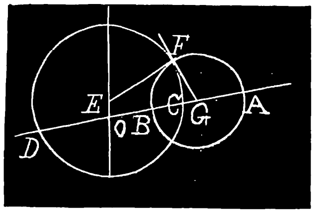
Also solved by *G. B. M. ZERR*, and the *PROPOSER*.

140. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Professor of Mathematics, Central High School, Dallas, Tex.

Having given two points on a range and a point that bisects the distance between two other points that form an harmonic ratio with the given points, give, if possible, a geometrical construction for locating the other two points.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A, B be the two given points on the range, O the point bisecting the distance between the other two points. Through O draw OE perpendicular to ABO, and on AB as diameter describe a circle AFB. Draw EF tangent to AFB, and with E as center and a radius equal to EF describe a circle cutting AO in C and D. (F is the point of tangency of EF, and EF must be greater than EO). Then C, D are the two points required.



For $GF^2=AG^2=GC.GD$.

$\therefore AG:GC=GD:AG$.

But $AG+GC:AG-GC=GD+AG:GD-AG$.

$\therefore AC:CB=AD:BD$. Q. E. D.

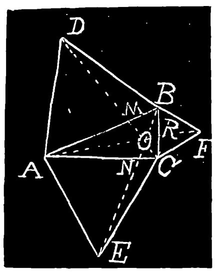
141. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

The equilateral triangle described on the hypotenuse of a right triangle is equivalent to the sum of the equilateral triangles described on the other two sides.

Prove without the aid of the famous Pythagorean proposition.

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and NELSON L. RORAY, Bridgeton, N. J.

Let ABC represent the right triangle, and D, E, F the vertices of the equilateral triangles constructed on the three sides.



It is seen at once that $\triangle ACF=\triangle BCE=\frac{1}{2}\triangle ABC$.

$\therefore \triangle ACF+\triangle BCE=\triangle ABC$.

$\triangle BDC=\triangle ABF$, $\triangle ADC=\triangle AEB$.

$\therefore \triangle ABD+\triangle ABC=\triangle ABF+\triangle AEB$.

$\triangle ABC=\triangle ACF+\triangle BCE$.

$\therefore \triangle ABD+2\triangle ABC=(\triangle ABF+\triangle ACF)+(\triangle AEB+\triangle BCE)$.

$\therefore \triangle ABD+2\triangle ABC=\triangle BCF+\triangle ABC+\triangle ACE+\triangle ABC$.